Standarddatenberechnung in Flexiblen Mehrkörpersystemen

Massenmatrix

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} \int_{\Omega_0} \mathbf{E}\,\mathrm{dm} & \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} & \int_{\Omega_0} \mathbf{\Phi}\,\mathrm{dm} \\ & \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}\tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} & \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}\mathbf{\Phi}\,\mathrm{dm} \\ \mathrm{sym.} & & \int_{\Omega_0} \mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}\,\mathrm{dm} \end{bmatrix}$$

Masse

$$\int_{\Omega_0} \mathbf{E}\,\mathrm{dm} = \begin{bmatrix} \mathrm{m} & 0 & 0 \\ 0 & \mathrm{m} & 0 \\ 0 & 0 & \mathrm{m} \end{bmatrix}$$

Schwerpunkt

$$\mathrm{m}\tilde{\mathbf{c}}^{\mathrm{T}} = \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} = \mathrm{m}\left(\widetilde{\mathbf{c}_0 + \mathbf{c}_1(\mathbf{q})}\right)^{\mathrm{T}}$$

$$\mathbf{c}_0 = \frac{1}{\mathrm{m}} \int_{\Omega_0} \mathbf{R}_{\mathrm{RP}}\,\mathrm{dm}$$

$$\mathbf{c}_1(\mathbf{q}) = \frac{1}{\mathrm{m}} \int_{\Omega_0} \mathbf{\Phi}\,\mathrm{dm}\,\mathbf{q} = \frac{1}{\mathrm{m}}\mathbf{C1}\,\mathbf{q}$$

Trägheitstensor

$$\mathbf{I}(\mathbf{q}) = \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}\tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} = \mathbf{I}_0 + \mathbf{I}_1(\mathbf{q}) + \mathbf{I}_2(\mathbf{q})$$

$$\mathbf{I}_0 = \int_{\Omega_0} \tilde{\mathbf{R}}_{\mathrm{RP}}\tilde{\mathbf{R}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm}$$

$$\mathbf{I}_1(\mathbf{q}) = \int_{\Omega_0} \left(\tilde{\mathbf{R}}_{\mathrm{RP}}(\widetilde{\mathbf{\Phi}\mathbf{q}})^{\mathrm{T}} + (\widetilde{\mathbf{\Phi}\mathbf{q}})\tilde{\mathbf{R}}_{\mathrm{RP}}^{\mathrm{T}}\right)\mathrm{dm} = -\sum_{\mathrm{l}=1}^{\mathrm{n_q}} \left(\mathbf{C4}_{\mathrm{l}} + \mathbf{C4}_{\mathrm{l}}^{\mathrm{T}}\right)\mathrm{q}_{\mathrm{l}}$$

$$\mathbf{I}_2(\mathbf{q}) = \int_{\Omega_0} (\widetilde{\mathbf{\Phi}\mathbf{q}})(\widetilde{\mathbf{\Phi}\mathbf{q}})^{\mathrm{T}}\,\mathrm{dm}$$

($\mathbf{I}_2$ entfällt bei Linearisierung in den Verformungskoordinaten)

Kopplungsterme - Translation

$$\mathbf{C}_\mathrm{t} = \int_{\Omega_0} \mathbf{\Phi}^\mathrm{T}\,\mathrm{dm} = \mathbf{C1}^\mathrm{T}$$

Kopplungsterme - Rotation

$$\mathbf{C}_\mathrm{r}(\mathbf{q}) = \int_{\Omega_0} \mathbf{\Phi}^\mathrm{T}\tilde{\mathbf{r}}_\mathrm{RP}^\mathrm{T}\,\mathrm{dm} = \mathbf{C}_\mathrm{r0} + \mathbf{C}_\mathrm{r1}(\mathbf{q})$$

$$\mathbf{C}_\mathrm{r0} = \int_{\Omega_0} \mathbf{\Phi}^\mathrm{T}\tilde{\mathbf{R}}_\mathrm{RP}^\mathrm{T}\,\mathrm{dm} = \mathbf{C2}^\mathrm{T}$$

$$\mathbf{C}_\mathrm{r1}(\mathbf{q}) = \int_{\Omega_0} \mathbf{\Phi}^\mathrm{T}(\widetilde{\mathbf{\Phi}\mathbf{q}})^\mathrm{T}\,\mathrm{dm} = \sum_{\mathrm{l}=1}^{\mathrm{n_q}} \mathbf{C5}_\mathrm{l}^\mathrm{T}\mathrm{q_l}$$

Massenmatrix des elastischen Subsystems

$$\mathbf{M}_\mathrm{e} = \int_{\Omega_0} \mathbf{\Phi}^\mathrm{T}\mathbf{\Phi}\,\mathrm{dm} = \mathbf{C3}_{11} + \mathbf{C3}_{22} + \mathbf{C3}_{33}$$

Generalisierte Trägheitskräfte

$$\begin{bmatrix}\mathbf{h}_{\omega\mathrm{t}}\\ \mathbf{h}_{\omega\mathrm{r}}\\ \mathbf{h}_{\omega\mathrm{e}}\end{bmatrix} = \begin{bmatrix} \int_{\Omega_0} \tilde{\boldsymbol{\omega}}_\mathrm{IR}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\mathbf{r}_\mathrm{RP} + 2\tilde{\boldsymbol{\omega}}_\mathrm{IR}\dot{\mathbf{r}}_\mathrm{RP}\,\mathrm{dm} \\ -\int_{\Omega_0} \tilde{\mathbf{r}}_\mathrm{RP}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\mathbf{r}_\mathrm{RP} + 2\tilde{\mathbf{r}}_\mathrm{RP}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\dot{\mathbf{r}}_\mathrm{RP}\,\mathrm{dm} \\ \int_{\Omega_0} \mathbf{\Phi}^\mathrm{T}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\mathbf{r}_\mathrm{RP} + 2\mathbf{\Phi}^\mathrm{T}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\dot{\mathbf{r}}_\mathrm{RP}\,\mathrm{dm} \end{bmatrix}$$

Anteile aus der Translation

$$\mathbf{h}_{\omega\mathrm{t}} = 2\,\tilde{\boldsymbol{\omega}}_\mathrm{IR}\mathbf{C1}\,\dot{\mathbf{q}} + \mathrm{m}\,\tilde{\boldsymbol{\omega}}_\mathrm{IR}\tilde{\boldsymbol{\omega}}_\mathrm{IR}\mathbf{c}$$

Anteile aus der Rotation

$$\mathbf{h}_{\omega\mathrm{r}} = \tilde{\boldsymbol{\omega}}_\mathrm{IR}\mathbf{I}\boldsymbol{\omega}_\mathrm{IR} + \sum_{\mathrm{l}=1}^{\mathrm{n_q}} \mathbf{G}_\mathrm{rl}(\mathbf{q})\dot{\mathrm{q}}_\mathrm{l}\boldsymbol{\omega}_\mathrm{IR}$$

$$\mathbf{G}_\mathrm{rl}(\mathbf{q}) = -2\,\mathbf{C4}_\mathrm{l} - 2\sum_{\mathrm{k}=1}^{\mathrm{n_q}} \mathbf{C6}_\mathrm{kl}\mathrm{q_k}$$

Anteile aus den elastischen Verformungen

$$\mathbf{h}_{\omega e} = \begin{bmatrix} \boldsymbol{\omega}_{IR}^T \mathbf{O}_e^1 \boldsymbol{\omega}_{IR} \\ \vdots \\ \boldsymbol{\omega}_{IR}^T \mathbf{O}_e^{n_q} \boldsymbol{\omega}_{IR} \end{bmatrix} + \sum_{l=1}^{n_q} \mathbf{G}_{el} \dot{q}_l \boldsymbol{\omega}_{IR}$$

$$\mathbf{O}_e^k = \int_{\Omega_0} \left(\tilde{\mathbf{R}} \tilde{\boldsymbol{\Phi}}_{*k}\right)^T dm + \sum_{l=1}^{n_q} \int_{\Omega_0} \tilde{\boldsymbol{\Phi}}_{*k} \tilde{\boldsymbol{\Phi}}_{*l} \, dm \, q_l = \mathbf{C4}_k^T + \sum_{l=1}^{n_q} \mathbf{C6}_{kl} q_l$$

$$\mathbf{G}_{el} = 2 \int_{\Omega_0} \left(\tilde{\boldsymbol{\Phi}}_{*l} \boldsymbol{\Phi}\right)^T dm = 2 \, \mathbf{C5}_l^T$$

Neben der Berechnung von m, $\mathbf{c}_0$ und $\mathbf{I}_0$ müssen die folgenden Integrale für eine effiziente Auswertung der Bewegungsgleichungen im Vorfeld der Zeitintegration berechnet werden:

Integral	Dimension
$\mathbf{C1} = \int_{\Omega_0} \boldsymbol{\Phi} \, dm$	$[3 \times n_q]$
$\mathbf{C2} = \int_{\Omega_0} \tilde{\mathbf{R}} \boldsymbol{\Phi} \, dm$	$[3 \times n_q]$
$\mathbf{C3}_{\alpha\beta} = \int_{\Omega_0} \boldsymbol{\Phi}_{\alpha *}^T \boldsymbol{\Phi}_{\beta *} \, dm$	$[n_q \times n_q]$
$\mathbf{C4}_l = \int_{\Omega_0} \tilde{\mathbf{R}} \tilde{\boldsymbol{\Phi}}_{*l} \, dm$	$[3 \times 3]$
$\mathbf{C5}_l = \int_{\Omega_0} \tilde{\boldsymbol{\Phi}}_{*l} \boldsymbol{\Phi} \, dm$	$[3 \times n_q]$
$\mathbf{C6}_{kl} = \int_{\Omega_0} \tilde{\boldsymbol{\Phi}}_{*k} \tilde{\boldsymbol{\Phi}}_{*l} \, dm$	$[3 \times 3]$

Hinweise zur Notation:

$l, k = 1, \ldots n_q$

$\alpha, \beta = 1, 2, 3$

$\boldsymbol{\Phi}_{\alpha *}$: α-te Zeile von $\boldsymbol{\Phi}$

$\boldsymbol{\Phi}_{*l}$: l-te Spalte von $\boldsymbol{\Phi}$