



EXERCISE SHEET 3

Exercise 4: System Response of Original and Reduced System

Consider the following dynamical system

$$\begin{aligned} \mathbf{M}_e \cdot \ddot{\mathbf{q}}(t) + \mathbf{D}_e \cdot \dot{\mathbf{q}}(t) + \mathbf{K}_e \cdot \mathbf{q}(t) &= \mathbf{B}_e \cdot \mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}_e \cdot \mathbf{q}(t) \end{aligned}$$

and the corresponding Petrov-Galerkin¹ reduced system with the projection matrix \mathbf{V} .

We provide you with unfinished code (`exercise04.m`) and all matrices (`exercise04_matrices.mat`). Add the following features to the unfinished code to complete the comparison of the original and reduced system:

- (a) Write the function `calculateSystemResponse` with the following input: matrices \mathbf{M}_e , \mathbf{D}_e , \mathbf{K}_e , \mathbf{B}_e , \mathbf{C}_e and an array of imaginary points \mathbf{s} where the transfer matrix should be calculated. The first output is the transfer matrix \mathbf{H}_N calculated at the points \mathbf{s} and the second output $\mathbf{H}_N_{\text{norm}}$ is the Frobenius norm of each transfer matrix.

The 7th input argument is an optional projection matrix \mathbf{V} . If it is given, the transfer matrix of the corresponding Petrov-Galerkin reduced system is calculated.

- (b) Calculate the relative error $\epsilon_{\text{F}}^{\text{rel}}$ in the Frobenius norm and plot it in a new figure with a linear scale for the abscissa and a logarithmic scale for the ordinate in the range 1 Hz to 10 kHz.
- (c) Calculate the \mathcal{H}_2 and \mathcal{H}_∞ norm of the error system. You can use the property $\mathbf{H}(-i\omega) = \overline{\mathbf{H}(i\omega)}$ which implies $\|\mathbf{H}(-i\omega)\|_{\text{F}} = \|\mathbf{H}(i\omega)\|_{\text{F}}$ and $\|\mathbf{H}(-i\omega)\|_2 = \|\mathbf{H}(i\omega)\|_2$. Therefore, you only need the already calculated transfer matrices since $f_{\text{max}} = 10 \text{ kHz}$ is assumed to be almost infinity.

Exercise 5: Helix

Consider a helix parameterized by

$$\mathbf{h}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ t \end{pmatrix}.$$

After projecting it with the (unknown) oblique projection matrix $\mathbf{P} \in \mathbb{R}^{3 \times 3}$, you will get a new curve

$$\mathbf{x}(t) = \mathbf{P} \cdot \mathbf{h}(t).$$

Figure `exercise05.fig` shows this curve $\mathbf{x}(t)$. What is the matrix \mathbf{P} and the domain of t ? Describe in a few sentences how you came up with this educated guess.

Notes

- The homework is due on Sunday night December 17th, 2023 at 11:59 p.m.
- All files can be found on ILIAS.

¹c.f. exercise 3 (f)