Praktische Lösung des Eigenwertproblems

Unterteilung der Eigenwertlösung in elementare Grundaufgaben

Steigerung von Effizienz und Kondition durch Umformulierung des Eigenwertproblems mit Ähnlichkeitstransformationen:

```fortran
subroutine rg(nm,n,a,wr,wi,z,iv1,fv1,ierr)
  c
  c  this subroutine calls the recommended sequence of
  c  subroutines from the eigensystem subroutine package (eispack)
  c  to find the eigenvalues and eigenvectors
  c  of a real general matrix.
  c
  on input
  c  nm  must be set to the row dimension of the two-dimensional
  c  array parameters as declared in the calling program
  c  dimension statement.
  c  n  is the order of the matrix a.
  c  a  contains the real general matrix.
  c
  on output
  c  wr and wi contain the real and imaginary parts,
  c  respectively, of the eigenvalues. complex conjugate
  c  pairs of eigenvalues appear consecutively with the
  c  eigenvalue having the positive imaginary part first.
  c  z contains the real and imaginary parts of the
  c  eigenvectors. if the j-th eigenvalue is real, the
  c  j-th column of z contains its eigenvector. if the j-th
  c  eigenvalue is complex with positive imaginary part, the
  c  j-th and (j+1)-th columns of z contain the real and
  c  imaginary parts of its eigenvector. the conjugate of this
  c  vector is the eigenvector for the conjugate eigenvalue.
  c  ierr  is an integer output variable set equal to an error
  c  completion code described in the documentation for hqr
  c  and hqr2. the normal completion code is zero.
  c  iv1 and fv1 are temporary storage arrays.
  c
  call balanc(nm,n,a,is1,is2,fv1)
  call elmhes(nm,n,is1,is2,a,iv1)
  call eltran(nm,n,is1,is2,a,iv1,z)
  call hqr2(nm,n,is1,is2,a,wr,wi,z,ierr)
  call balbak(nm,n,is1,is2,fv1,n,z)
  return
end
```
Balancieren

Skalieren des Eigenwertproblems zur Konditionsverbesserung durch Diagonalmatrizen:

Konzeption des Skalierens:

subroutine balanc(nm,n,a,low,igh,scale)
  c
  c     this subroutine balances a real matrix and isolates
c     eigenvalues whenever possible.
c  c
  c on input
  c     n is the order of the matrix.
c     a contains the input matrix to be balanced.
c  c on output
  c     a contains the balanced matrix.

Rücktransformation der Skalierung auf Ursprungsdarstellung nach der Lösung des Eigen-
wertproblems:

subroutine balbak(nm,n,low,igh,scale,m,z)
  c
  c     this subroutine forms the eigenvectors of a real general
c     matrix by back transforming those of the corresponding
c     balanced matrix determined by balanc.
c  c
  c on input
  c     n is the order of the matrix.
c     z contains the real and imaginary parts of the eigen-
c     vectors to be back transformed in its first m columns.
c  c on output
  c     z contains the real and imaginary parts of the
  c     transformed eigenvectors in its first m columns.
Reduktion auf obere Hessenberg–Form

Transformation auf obere Hessenberg–Form zur Reduktion des Rechenaufwands durch Ähnlichkeitstransformationen:

1) Permutationen
2) Eliminationen
3) Spiegelungen
4) Drehungen

```
subroutine elmhes(nm,n,low,igh,a,int)
  c
  c  given a real general matrix, this subroutine
  c  reduces a submatrix situated in rows and columns
  c  low through igh to upper hessenberg form by
  c  stabilized elementary similarity transformations.
  c
  c  on input
  c    n is the order of the matrix.
  c    a contains the input matrix.
  c  on output
  c    a contains the hessenberg matrix. the multipliers
  c      which were used in the reduction are stored in the
  c      remaining triangle under the hessenberg matrix.
```

```
subroutine eltran(nm,n,low,igh,a,int,z)
  c
  c  this subroutine accumulates the stabilized elementary
  c  similarity transformations used in the reduction of a
  c  real general matrix to upper hessenberg form by elmhes.
  c
  c  on input
  c    n is the order of the matrix.
  c    a contains the multipliers which were used in the
  c      reduction by elmhes in its lower triangle
  c      below the subdiagonal.
  c  on output
  c    z contains the transformation matrix produced in the
  c      reduction by elmhes.
```
Iterative Eigenwertlösung

Überführung des Eigenwertproblems in Rechtsdreiecksform durch

1) Jacobi–Verfahren
2) LR–Verfahren
3) QR–Verfahren

subroutine hqr2(nm,n,low,igh,h,wr,wi,z,ierr)

  this subroutine finds the eigenvalues and eigenvectors
  of a real upper hessenberg matrix by the qr method. the
  eigenvectors of a real general matrix can also be found
  if elmhes and eltran or orthes and ortran have
  been used to reduce this general matrix to hessenberg form
  and to accumulate the similarity transformations.

  on input
  n is the order of the matrix.
  h contains the upper hessenberg matrix.
  z contains the transformation matrix produced by eltran
  after the reduction by elmhes, or by ortran after the
  reduction by orthes, if performed. if the eigenvectors
  of the hessenberg matrix are desired, z must contain the
  identity matrix.

  on output
  wr and wi contain the real and imaginary parts,
  respectively, of the eigenvalues. the eigenvalues
  are unordered except that complex conjugate pairs
  of values appear consecutively with the eigenvalue
  having the positive imaginary part first. if an
  error exit is made, the eigenvalues should be correct
  for indices ierr+1,...,n.
  z contains the real and imaginary parts of the eigenvectors.
  if the i-th eigenvalue is real, the i-th column of z
  contains its eigenvector.
  if the i-th eigenvalue is complex
  with positive imaginary part, the i-th and (i+1)-th
  columns of z contain the real and imaginary parts of its
  eigenvector. the eigenvectors are unnormalized. if an
  error exit is made, none of the eigenvectors
  has been found.