Trajectory Tracking Control of a Very Flexible Robot Using a Feedback Linearization Controller and a Nonlinear Observer

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Abstract Flexible robots can be modeled as underactuated multibody systems since they generally have less control inputs than degrees of freedom for rigid body motion and deformation. The flexibilities must be taken into account in the control design. In order to obtain high performance in the end-effector trajectory tracking, an accurate and efficient nonlinear controller is required. In this paper, a nonlinear feedback controller based on the feedback linearization approach using all the states of the system is designed and carefully tested on a very flexible parallel lambda robot. The simulation and experimental results show that the end-effector tracks a trajectory with higher accuracy compared to previous works.

1 Introduction

Light-weight manipulators attract a lot of research interest because of their complementing advantages. The advantages of light-weight robots include low energy usage, less mass, and often high working speeds. However, due to the light-weight design, the bodies have a significant flexibility which yields undesired deformations and vibrations. Therefore, the manipulators are modeled as a flexible multibody system and the flexibilities must be taken into account in the control design. The flexible system with significant deformations complicates the control design because there are more generalized coordinates than control inputs. In order to obtain a high performance in the end-effector trajectory tracking of a flexible manipulator, an accurate and efficient model and nonlinear controller is necessary. The difficulty of designing a nonlinear feedback controller with high performance for a highly flexible system is increased, when the controller does not have access to direct measurement of the end-effector and all the system states. To overcome this problem, an observer to estimate all the system states and end-effector position is required. Finally, based on the estimated states of the system, a nonlinear feedback controller can be designed.

In previous works on the lambda robot, the feedback controllers just used the measurable states of the system, e.g. the position and velocity of the actuators and the deformation of the flexible link. Using these outputs, a linear controller was designed for the position and velocity of the actuators, see Burkhardt et al. (2014). Also, an additional gain scheduling for the linear controller of the actuators and a curvature controller based on the strain of the flexible link were designed by Morlock et al. (2016).

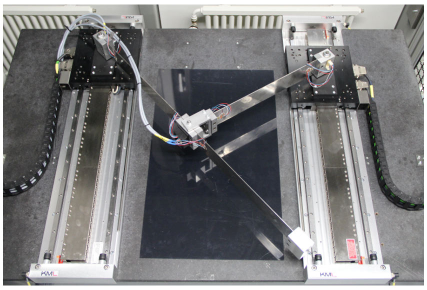
The novelty of this work is, that a nonlinear feedback controller for high-speed trajectory tracking of a very flexible parallel lambda robot is designed based on the feedback linearization approach and all the estimated states by the nonlinear observer. Hence, a nonlinear observer to estimate all the states is used, see Ansarieshlaghi and Eberhard (2017), to make nonlinear feedback control possible. Then, a feedback linearization control approach, see Khalil (2002), is utilized to track a trajectory by the flexible robot in real-time.

The nonlinear feedback controller using the nonlinear observer is simulated and carefully tested on the lambda robot hardware. Experimental and simulation results of the designed feedback linearization controller on the lambda robot show that the end-effector tracks a trajectory with high accuracy and the tracking performance is drastically improved compared to previous works, see Morlock et al. (2016).

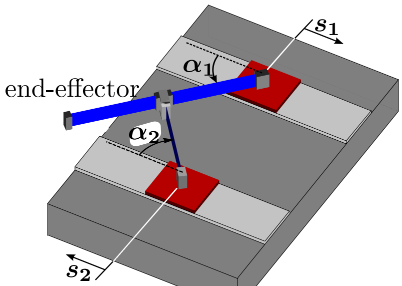
The paper is organized as follows: Section 2 includes the modeling of the flexible parallel lambda robot. Section 3 includes the description of the nonlinear control, i.e. the feedforward controller, nonlinear observer, and feedback controller. In Section 4, the proposed nonlinear controller is tested on the simulated model and the hardware and the results are discussed.

2 Flexible Lambda Robot Modelling

The used lambda robot has highly flexible links. The end of the short link is connected in the middle of the long link using rigid bodies. This robot has two prismatic actuators connecting the links to the ground. The links are connected using passive revolute joints to the linear actuators. Another revolute joint is used to connect the short link and the middle of the long link. An additional rigid body is attached to the free end of the long link as an end-effector. The drive positions and velocities are measured with optical encoders. Three full Wheatstone bridge strain gauges are attached to measure the deformation of the flexible long link. The lambda robot configuration shown in Figure 1a has been built in hardware, see Burkhardt et al. (2014), at the Institute of Engineering and Computational Mechanics of the University of Stuttgart.



(a) Hardware of lambda robot



(b) Model of lambda robot

Figure 1: Lambda robot

The modeling process of the flexible manipulator with λ configuration can be separated into three major steps. First, the flexible components of the system are modeled separately with the linear finite element method in the commercial finite element code *ANSYS*. Next, in order to control the λ robot, the deformation degrees of freedom of the flexible bodies shall be decreased and thus model order reduction is utilized. Then, all the rigid and flexible parts are combined to a flexible multibody system with a kinematic loop.

The equation of motion with a kinematic loop constraint for the flexible parallel manipulator, using the generalized coordinates $\boldsymbol{q} \in \mathbb{R}^5$ is

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{k}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{g}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{B}(\boldsymbol{q})\boldsymbol{u} + \boldsymbol{C}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\lambda}\,, \tag{1a}$$

$$\boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{0}\,. \tag{1b}$$

The symmetric, positive definite mass matrix $\boldsymbol{M} \in \mathbb{R}^{5\times5}$ depends on the joint positions, angles, and the elastic coordinate. The vector $\boldsymbol{k} \in \mathbb{R}^5$ contains the generalized centrifugal, Coriolis and Euler forces, and $\boldsymbol{g} \in \mathbb{R}^5$ includes the vector of applied forces and inner forces due to the body elasticity. The input matrix $\boldsymbol{B} \in \mathbb{R}^{5\times2}$ maps the input vector $\boldsymbol{u} \in \mathbb{R}^2$ to the system. The constraint equations are defined by $\boldsymbol{c} \in \mathbb{R}^2$. The Jacobian matrix of the constraint $\boldsymbol{C} = \partial\left(\boldsymbol{c}(\boldsymbol{q})\right)/\partial\boldsymbol{q} \in \mathbb{R}^{2\times5}$ maps the reaction force $\boldsymbol{\lambda} \in \mathbb{R}^2$ due to the kinematic loop. The mechanical model of the flexible lambda robot is shown in Figure 1b. The robot is modeled with four rigid degrees of freedom that is shown in in Figure 1b, and one elastic coordinate as $\boldsymbol{q}_e$.

3 Nonlinear Controller

The lambda robot control strategy is separated into feedforward and feedback control parts. In the feedforward control part, the desired trajectories for system states are calculated from a two-point boundary value problem offline while the flexible multibody system is a non-minimum phase system with internal dynamics. The results of the feedforward part are the desired values for the feedback control part. The real-time control part is divided into a nonlinear observer to estimate all the system states and a feedback linearization controller of the lambda robot. Figure 2 shows the nonlinear controller structure of the lambda robot.

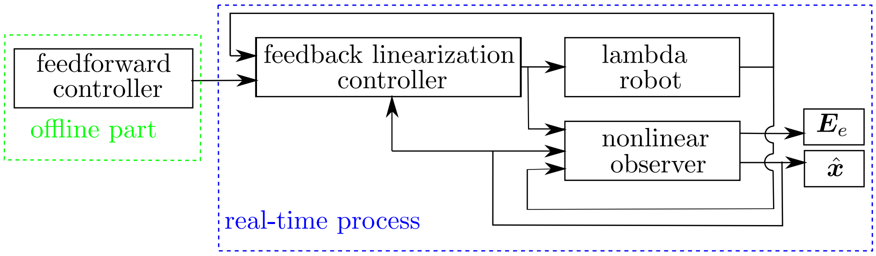


Figure 2: Control block diagram

3.1 Feedforward Controller

The feedforward controller is obtained by solving a two-point boundary value problem for the exact model inversion of the complete dynamical model of the multibody system. To force the end-effector to track a trajectory, an additional constraint equation, the so-called servo constraint, see Blajer and Kolodziejczyk (2004), is augmented to the equation of motion (1). The equation of motion with the new servo constraint can be written as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{k}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{g}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{B}(\boldsymbol{q})\boldsymbol{u} + \boldsymbol{C}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\lambda} \; , \tag{2a}$$

$$\boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{0} \; , \tag{2b}$$

$$\boldsymbol{s}(t, \boldsymbol{q}) = \boldsymbol{0} \; . \tag{2c}$$

The solution of the boundary value problem is computed in *Matlab* using the solver *bvp5c*. The set values of the dependent coordinates, the independent coordinates, the inputs of the system, and the Lagrange multipliers are obtained from the constraint equations on the position, velocity, and acceleration level.

3.2 Nonlinear Observer

A nonlinear high gain observer for the lambda robot was designed to estimate the states and the end-effector position, see Ansarieshlaghi and Eberhard (2017). For state estimation, the dynamics description of the lambda robot in state space form Equation (1a) can be written as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}), \tag{3a}$$

$$\boldsymbol{y} = \boldsymbol{W}\boldsymbol{x} = [s_1 \quad s_2 \quad \dot{s}_1 \quad \dot{s}_2 \quad q_e]^{\mathrm{T}}, \tag{3b}$$

where $\boldsymbol{y}$ are the system outputs, $\boldsymbol{A}$ is a constant matrix, and $\boldsymbol{f}(\boldsymbol{x})$ is a nonlinear function that are defined as follows

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0}_{5\times5} & \boldsymbol{I}_{5\times5} \\ \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} \end{bmatrix}, \tag{4a}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{M}^{-1}(\boldsymbol{x})(-\boldsymbol{k}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{C}^T(\boldsymbol{x})\boldsymbol{\lambda} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}) \end{bmatrix}. \tag{4b}$$

State estimation is done for the system in Equation (3) using a high gain observer approach for the nonlinear system, see Primbs (1996) or Khalil (2008). The dynamics of the proposed observer is formulated as

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{L}(\hat{\boldsymbol{y}} - \boldsymbol{y}), \tag{5a}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{W}\hat{\boldsymbol{x}}, \tag{5b}$$

where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are the estimated states and outputs of the observed system, respectively. Therefore, the observer gain $\boldsymbol{L} \in \mathbb{R}^{10\times5}$ shall be designed such that the estimated states converge to the real system states. The observation error is calculated with the real system states in Equation (3) and the estimated states from the observer in Equation (5) as $\boldsymbol{e} = \hat{\boldsymbol{x}} - \boldsymbol{x}$.

Using the Lyapunov function and the Lipschitz condition one can show that the estimation error converges to zero. That means that the estimated states converge to the real system measurements. The values $\hat{\boldsymbol{x}}$ and $\boldsymbol{E}_e$ are outputs of the nonlinear observer, i.e. the estimated states and end-effector positions. The estimated states are fed to the feedback controller part in order to calculate the input using a feedback linearization approach.

3.3 Feedback Linearization Controller

In order to design a nonlinear feedback controller for the system in Equation (1a), the system dynamics can be written in state space as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{5\times5} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \underbrace{\boldsymbol{M}^{-1}(\boldsymbol{x})(-\boldsymbol{k}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{C}^T(\boldsymbol{x})\boldsymbol{\lambda})}_{\boldsymbol{n}(\boldsymbol{x})} + \underbrace{\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})}_{\boldsymbol{G}(\boldsymbol{x})}\boldsymbol{u} \end{bmatrix}, \tag{6a}$$

$$\boldsymbol{y}_{fl} = \boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}, \tag{6b}$$

where $\boldsymbol{y}_{fl} \in \mathbb{R}^2$ is the output of the lambda robot and the loop closing constrained must be obeyed. The vector $\boldsymbol{h} \in \mathbb{R}^2$ is a function of the system states. For input-output linearization, only two outputs of the lambda robot such as the actuator positions are used. The feedback linearization controller cancels the nonlinear part of the robot dynamics using the estimated states by a nonlinear observer and then it controls the linearized system by a linear controller. The control law is then obtained for the lambda robot using the estimated states as

$$\boldsymbol{u} = \boldsymbol{P}^{-1}(\hat{\boldsymbol{x}})(\boldsymbol{v} - \boldsymbol{b}(\hat{\boldsymbol{x}})). \tag{7}$$

The linear part of controller $\boldsymbol{v} \in \mathbb{R}^2$ is used for the linearized system. The matrix $\boldsymbol{P} \in \mathbb{R}^2$ is the decoupling matrix, and $\boldsymbol{b} \in \mathbb{R}^2$ is the vector of the nonlinear part of the dynamics. In the linear part of the controller, a PI controller for position and a PI controller for velocity are designed. The matrix $\boldsymbol{P}$ and vector $\boldsymbol{b}$ are nonlinear functions of the system states

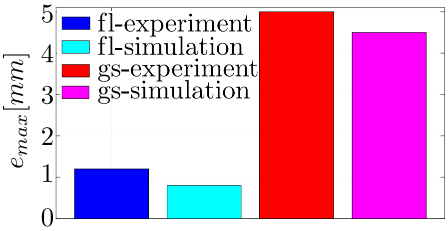
$$\boldsymbol{P}(\hat{\boldsymbol{x}}) = \begin{bmatrix} L_{G_1} L_n^{r_1-1} h_1(\boldsymbol{x}) & L_{G_2} L_n^{r_1-1} h_1(\boldsymbol{x}) \\ L_{G_1} L_n^{r_2-1} h_2(\boldsymbol{x}) & L_{G_2} L_n^{r_2-1} h_2(\boldsymbol{x}) \end{bmatrix}, \quad \boldsymbol{b}(\hat{\boldsymbol{x}}) = \begin{bmatrix} L_n^{r_1} h_1(\boldsymbol{x}) \\ L_n^{r_2} h_2(\boldsymbol{x}) \end{bmatrix}. \tag{8}$$

Here, L_{G_i} is the Lie derivative respect to the matrix $\boldsymbol{G}$ and $L_n^{r_1-1}$ is the Lie derivative respect to the vector $\boldsymbol{n}$ with relative degree $r_1 - 1$. For the lambda robot, the relative degree of the system outputs is $r_1 = r_2 = 2$.

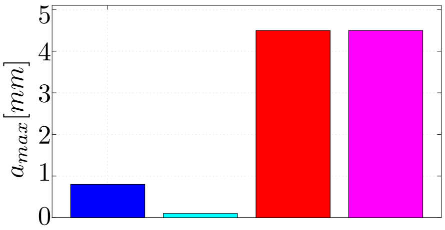
4 Simulation and Experimental Results

To validate the designed nonlinear feedback controller, the end-effector tracks a line trajectory and a camera records a movie during the trajectory tracking. The recorded movie is used for offline validation.

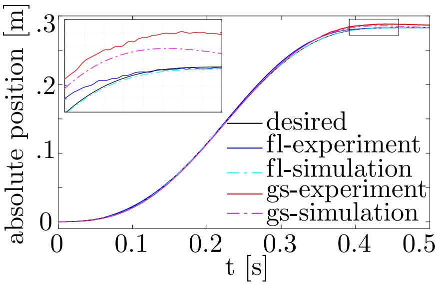
Figure 3 shows the simulation and experimental results of the designed nonlinear feedback controller and its comparison to the gain scheduling controller that was presented in Morlock et al. (2016).



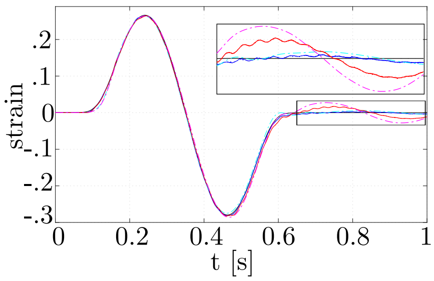
(a) Maximum tracking error



(b) Maximum oscillation amplitude



(c) Absolute position of end-effector



(d) Strain of long flexible link

Figure 3: Simulation and experimental results for tracking a line trajectory using a feedback linearization controller (fl) and a gain-scheduling controller (gs).

The simulation and experimental results in Figure 3 show that the designed feedback linearization controller tracks the trajectory with high accuracy. The presented nonlinear controller based on the model and using the nonlinear observer in Ansarieshlaghi and Eberhard (2017) reduces the trajectory tracking error about 76% and the end-point error about 90% in simulation and experimental tests that are shown in Figures 3a and 3c. On the other hand, the oscillation amplitude in simulation and experiment is decreased about 79% as shown in Figures 3b and 3d.

Conclusion

In this paper, a nonlinear feedback controller was designed and applied experimentally to a very flexible multibody system. The feedback linearization controller obtains the system inputs based on the estimated states by the nonlinear observer. The feedback linearization controller cancels the nonlinear part of the system dynamics using the estimated states and then controls the linearized system with a linear controller. The experimental results for the very flexible parallel robot show that the feedback controller successfully tracks the desired trajectory with high accuracy. Experimental validation results demonstrate that the tracking error and oscillation amplitude drastically decrease compared to the gain scheduling approach.

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