Nonlinear Position Control of a Very Flexible Parallel Robot Manipulator

Peter Eberhard, Fatemeh Ansarieshlaghi

Institute of Engineering and Computational Mechanics
University of Stuttgart
Pfaffenwaldring 9, 70569 Stuttgart, Germany
[peter.eberhard, fatemeh.ansari]@itm.uni-stuttgart.de

EXTENDED ABSTRACT

1 Introduction

Flexible robots are examples of underactuated multibody systems since they generally have less control inputs than degrees of freedom for rigid body motion and deformation. In order to obtain high performance in the end-effector trajectory tracking of flexible manipulators, an accurate and efficient nonlinear feedforward and feedback controller are advantageous. In this contribution, a model based control is used to control a flexible multibody system. Therefore, the accuracy of the system model is important. Furthermore, the considered flexible manipulator has a significant flexibility which yields undesired deformations and vibrations. The manipulator shall be modeled as a flexible system and the flexibilities must be taken into account in the controller design. The flexible system with significant deformations complicates the controller design because there are more generalized coordinates than control inputs. In this paper, a nonlinear feedback controller based on the feedback linearization approach using the system states is designed and implemented on a very flexible parallel robot model. The results show that the end-effector tracks a trajectory with high accuracy.

2 Modeling of flexible robot

The modeling process of the flexible manipulator with λ configuration can be separated into three major steps. First, the flexible components of the system are modeled with the linear finite element method in the commercial finite element code *ANSYS*. Next, in order to control the λ robot, the degrees of freedom of the flexible bodies shall be decreased. Therefore, model order reduction is utilized. Then, all the rigid and flexible parts are modeled as a multibody system with a kinematic loop. The equation of motion with a kinematic loop constraint of the flexible parallel manipulator, using the generalized coordinates $\boldsymbol{q} \in \mathbb{R}^5$ is

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{k}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{g}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{B}(\boldsymbol{q})\boldsymbol{u} + \boldsymbol{C}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\lambda} \;, \tag{1a}$$

$$\boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{0} \,. \tag{1b}$$

The symmetric, positive definite mass matrix $\boldsymbol{M} \in \mathbb{R}^{5\times 5}$ depends on the joint angles and the elastic coordinates. The vector $\boldsymbol{k} \in \mathbb{R}^5$ contains the generalized centrifugal, Coriolis and Euler forces, and $\boldsymbol{g} \in \mathbb{R}^5$ includes the vector of applied forces and inner forces due to the body elasticity. The input matrix $\boldsymbol{B} \in \mathbb{R}^{5\times 2}$ maps the input vector $\boldsymbol{u} \in \mathbb{R}^2$ to the system. The constraint equations are defined by $\boldsymbol{c} \in \mathbb{R}^2$. The Jacobian matrix of the constraint $\boldsymbol{C} = \partial\left(\boldsymbol{c}(\boldsymbol{q})\right)/\partial \boldsymbol{q} \in \mathbb{R}^{2\times 5}$ maps the reaction force $\boldsymbol{\lambda} \in \mathbb{R}^2$ due to the kinematic loop. The flexible lambda robot in hardware and its mechanical model are shown in Figure 1 left, see also [1, 2].

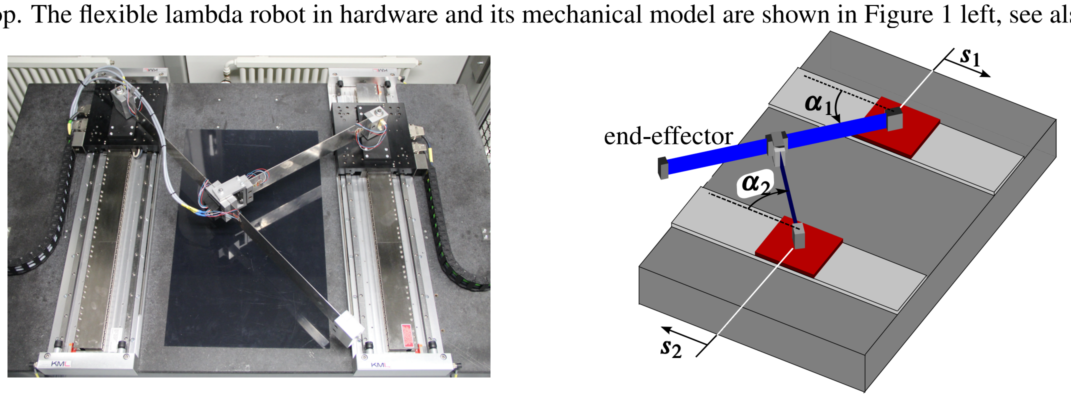

Figure 1: Lambda robot, mechanical setup of the robot and simulation model of flexible parallel robot

While the system has a kinematics loop as a constraint, the system coordinates derivation $\ddot{\boldsymbol{q}}$ can be divided to dependent $\ddot{\boldsymbol{q}}_{\mathrm{d}}$ and independent $\ddot{\boldsymbol{q}}_{\mathrm{i}}$ coordinates and the system constraints can be written as

$$\ddot{\boldsymbol{c}} = \boldsymbol{C}\ddot{\boldsymbol{q}} + \boldsymbol{c}'' = \boldsymbol{C}_{\mathrm{i}}\ddot{\boldsymbol{q}}_{\mathrm{i}} + \boldsymbol{C}_{\mathrm{d}}\ddot{\boldsymbol{q}}_{\mathrm{d}} + \boldsymbol{c}'' \tag{2}$$

where $\boldsymbol{C}_{\mathrm{d}}$ and $\boldsymbol{C}_{\mathrm{i}}$ are dependent and independent parts of the Jacobian matrix of the constraint and $\boldsymbol{c}''$ presents local accelerations due to the constraints. Based on Equation (2), the system coordinates can be formulated by

$$\ddot{\boldsymbol{q}} = \begin{bmatrix} \ddot{\boldsymbol{q}}_{\mathrm{i}} \\ \ddot{\boldsymbol{q}}_{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{3\times 3} \\ -\boldsymbol{C}_{\mathrm{d}}^{-1}\boldsymbol{C}_{\mathrm{i}} \end{bmatrix} \ddot{\boldsymbol{q}}_{\mathrm{i}} + \begin{bmatrix} \boldsymbol{0}_{3\times 3} \\ -\boldsymbol{C}_{\mathrm{d}}^{-1}\boldsymbol{c}'' \end{bmatrix} = \boldsymbol{J}_{\mathrm{c}}\ddot{\boldsymbol{q}}_{\mathrm{i}} + \boldsymbol{b}''. \tag{3}$$

Finally, by left-side multiplication with the transposed of the Jacobian matrix $\boldsymbol{J}_\mathrm{c}$ and replacing $\ddot{\boldsymbol{q}}$ by Equation (3), the equation of motion (1a) can be determined, see [1], by

$$\bar{\boldsymbol{M}}\ddot{\boldsymbol{q}}_\mathrm{i} = -\bar{\boldsymbol{k}} + \bar{\boldsymbol{g}} + \bar{\boldsymbol{B}}\boldsymbol{u} = \bar{\boldsymbol{f}} + \bar{\boldsymbol{B}}\boldsymbol{u}. \tag{4}$$

3 Control of flexible lambda robot

The lambda robot control is separated into feedforward and feedback control parts. In the feedforward control part, the desired trajectories for the system states are calculated using a two-point boundary value problem while the flexible multibody system is a non-minimum phase system with internal dynamics [1]. The feedback control part computes the lambda robot input using the nonlinear dynamics of the robot and the measured states based on the feedback linearization method in [3].

In order to design a nonlinear controller for the system in Equation (4), the system dynamics can be written in the state space

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{q}}_\mathrm{i} \\ \ddot{\boldsymbol{q}}_\mathrm{i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{3\times 3} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \bar{\boldsymbol{M}}^{-1}(\bar{\boldsymbol{f}} + \bar{\boldsymbol{B}}\boldsymbol{u}) \end{bmatrix}. \tag{5}$$

The control law is then obtained for the lambda robot as

$$\boldsymbol{u} = \bar{\boldsymbol{B}}^{-1}(-\bar{\boldsymbol{f}} + \bar{\boldsymbol{M}}(\boldsymbol{K}_\mathrm{P}\boldsymbol{e} + \boldsymbol{K}_\mathrm{D}\dot{\boldsymbol{e}})), \tag{6}$$

where $\boldsymbol{q}_\mathrm{id}$ is the desired value for $\boldsymbol{q}_\mathrm{i}$, consequently $\dot{\boldsymbol{q}}_\mathrm{id}$ is the desired value for $\dot{\boldsymbol{q}}_\mathrm{i}$ and the error and dynamics of error are calculated by $\boldsymbol{e} = \boldsymbol{q}_\mathrm{i} - \boldsymbol{q}_\mathrm{id}$ and $\dot{\boldsymbol{e}} = \dot{\boldsymbol{q}}_\mathrm{i} - \dot{\boldsymbol{q}}_\mathrm{id}$. The desired values depend on the desired trajectory of the end-effector and can be computed via the feedforward part and $\boldsymbol{q}_\mathrm{id}$ and $\dot{\boldsymbol{q}}_\mathrm{id}$ can be set. The matrices $\boldsymbol{K}_\mathrm{P}$ and $\boldsymbol{K}_\mathrm{D}$ correspond to the weighting of the state feedback error and can be designed via the LQR method or tuned by hand. The inverse of the input matrix $\bar{\boldsymbol{B}}$ is not so straightforward to calculate since it is not of full row rank. Therefore, the existing left-inverse is used as a pseudo-inverse to yield $\bar{\boldsymbol{B}}\bar{\boldsymbol{B}}^{-1} = \boldsymbol{I}$.

4 Simulation results

To validate the designed nonlinear feedback controller, the end-effector tracks a line and an eight shape trajectory. Figure 2 shows the simulation results of the designed nonlinear feedback controller based on the reduced model and its comparison to the nonlinear feedback controller that was presented in [4]. The simulation results in Figure 2 show that the designed feedback linearization controller based on the reduced model tracks the trajectory with higher accuracy and less oscillation amplitude than the nonlinear controller in [4].

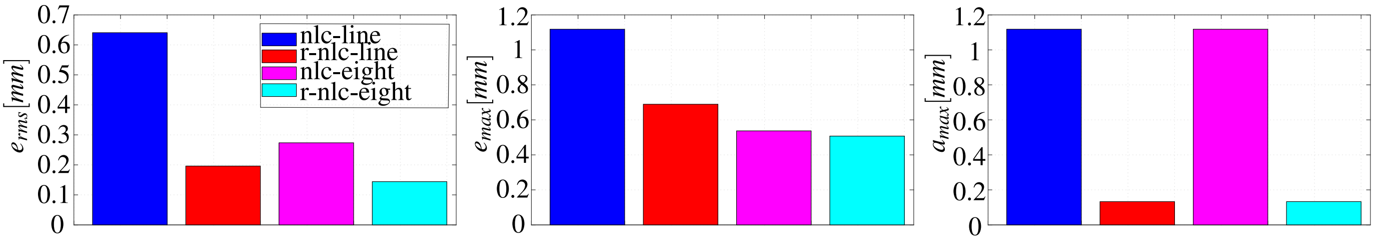

Figure 2: Comparing RMS and maximum error as well as maximum oscillation amplitude for tracking a line and an eight shape trajectory using two controllers, i.e., the nonlinear controller (nlc) and the reduced nonlinear controller (r-nlc).

Acknowledgements

This research was partially supported by the German Research Foundation within the Cluster of Excellence in Simulation Technology SimTech at the University of Stuttgart. The authors appreciate these discussions.

References

[1] M. Burkhardt, P. Holzwarth, R. Seifried, Inversion based trajectory tracking control for a parallel kinematic manipulator with flexible links. Proceedings of the 11th International Conference on Vibration Problems, Lisbon, Portugal, 2013.

[2] F. Ansarieshlaghi, P. Eberhard, Design of a nonlinear observer for a very flexible parallel robot. Proceedings of the 7th GACM Colloquium on Computational Mechanics, Stuttgart, Germany, 2017.

[3] H.K. Kalil, Nonlinear Systems, 3rd edition, Prentice-Hall, Upper Saddle River, New Jersey, 2002.

[4] F. Ansarieshlaghi, P. Eberhard, Trajectory tracking control of a very flexible robot using a feedback linearization controller and a nonlinear observer. Proceedings of the 22nd CISM IFToMM Symposium on Robot Design, Dynamics and Control, Rennes, France, 2018.