Design of a Feedback Linearization Controller for a Flexible Robot

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Summary: Flexible robots are examples of underactuated multibody systems since they generally have less control inputs than degrees of freedom for rigid body motion and deformation. In order to obtain high performance in the end-effector trajectory tracking of flexible manipulators, an accurate and efficient nonlinear feedforward controller is advantageous. This is then supplemented by an additional nonlinear feedback controller. In this paper, a nonlinear feedback controller based on the feedback linearization approach using the states of the system is designed and carefully tested on a very flexible parallel robot. The results show that the end-effector tracks a trajectory with higher accuracy compared to previous works.

1 Introduction

The considered flexible manipulator has a significant flexibility which yields undesired deformations and vibrations. Therefore, the manipulator is modeled as a flexible system and the flexibilities must be taken into account in the controller design. The flexible system with significant deformations complicates the control design because there are more generalized coordinates than control inputs. In this contribution, the control design of a parallel manipulator with flexible links is investigated. Hence, a model based control is used to control a flexible multibody system. Also, a nonlinear observer to estimate all the states is used [1] to make nonlinear feedback control possible. Then, a feedback linearization controller [2] is designed to control the flexible robot in real-time.

2 Modeling of flexible robot

The modeling process of the flexible manipulator with λ configuration can be separated into three major steps. First, the flexible components of the system are modeled with the linear finite element method in the commercial finite element code *ANSYS*. Next, in order to control the λ robot, the degrees of freedom of the flexible bodies shall be decreased. Therefore, model order reduction is utilized. Then, all the rigid and flexible parts are modeled as a multibody system with a kinematic loop.

The equation of motion with a kinematic loop constraint for the flexible parallel manipulator, using the generalized coordinates $\boldsymbol{q} \in \mathbb{R}^5$ is

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{k}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{g}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{B}(\boldsymbol{q})\boldsymbol{u} + \boldsymbol{C}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\lambda}\,, \tag{1a}$$

$$\boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{0}\,. \tag{1b}$$

The symmetric, positive definite mass matrix $\boldsymbol{M} \in \mathbb{R}^{5\times5}$ depends on the joint angles and the elastic coordinates. The vector $\boldsymbol{k} \in \mathbb{R}^5$ contains the generalized centrifugal, Coriolis and Euler forces, and $\boldsymbol{g} \in \mathbb{R}^5$ includes the vector of applied forces and inner forces due to the body elasticity. The input matrix $\boldsymbol{B} \in \mathbb{R}^{5\times2}$ maps the input vector $\boldsymbol{u} \in \mathbb{R}^2$ to the system. The constraint equations are defined by $\boldsymbol{c} \in \mathbb{R}^2$. The Jacobian matrix of the constraint $\boldsymbol{C} = \partial\left(\boldsymbol{c}(\boldsymbol{q})\right)/\partial\boldsymbol{q} \in \mathbb{R}^{2\times5}$ maps the reaction force $\boldsymbol{\lambda} \in \mathbb{R}^2$ due to the kinematic loop. The flexible lambda robot in hardware and its mechanical model are shown in Figure 1.

(a) Mechanical setup of the lambda robot. (b) Flexible parallel lambda robot

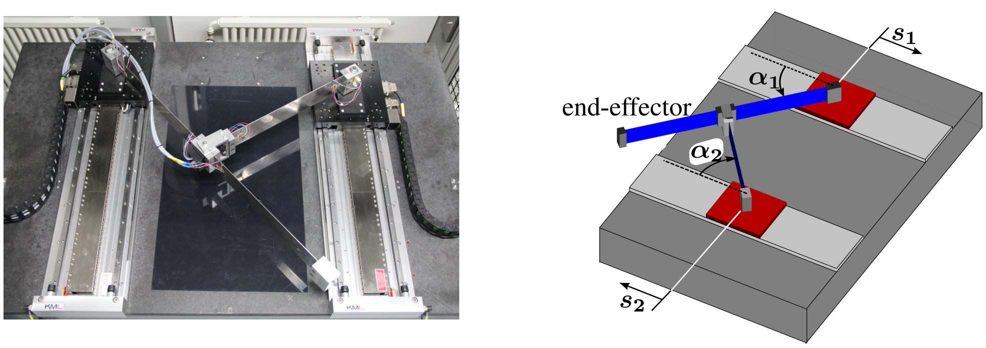

Figure 1: Lambda robot

3 Control of flexible lambda robot

The lambda robot control is separated into feedforward and feedback control parts. In the feedforward control part, the desired trajectories for the system states are calculated using a two-point boundary value problem while the flexible multibody system is a non-minimum phase system with internal dynamics [3]. The feedback control part computes the lambda robot input based on the nonlinear dynamics of the robot and the estimated states.

3.1 Feedback linearization controller

In order to design a nonlinear controller for the system in Eq. (1a), the system dynamics can be written in state space as follows

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{5\times 5} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \underbrace{\boldsymbol{M}^{-1}(\boldsymbol{x})(-\boldsymbol{k}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{C}^T(\boldsymbol{x})\boldsymbol{\lambda})}_{\boldsymbol{f}(\boldsymbol{x})} + \underbrace{\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})}_{\boldsymbol{G}(\boldsymbol{x})}\boldsymbol{u} \end{bmatrix}, \tag{2a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}, \tag{2b}$$

where $\boldsymbol{y} \in \mathbb{R}^2$ is the output of the lambda robot and the loop closing constrained must be obeyed. The vector $\boldsymbol{h} \in \mathbb{R}^2$ is a function of system states. The feedback linearization controller cancels the nonlinear part of the robot dynamics using the estimated states by a nonlinear observer and then it controls the linearized system by a linear controller. The control law is then obtained for the lambda robot as

$$\boldsymbol{u} = \boldsymbol{A}^{-1}(\boldsymbol{x})(\boldsymbol{v} - \boldsymbol{b}(\boldsymbol{x})). \tag{3}$$

Here, $\boldsymbol{v} \in \mathbb{R}^2$ is the linear part of controller for the linearized system, $\boldsymbol{A} \in \mathbb{R}^2$ is the decoupling matrix, and $\boldsymbol{b} \in \mathbb{R}^2$ is the vector of the nonlinear part of the dynamics. In the linear part of the controller, a $\boldsymbol{PI}$ controller for position and a $\boldsymbol{PI}$ controller for velocity are used. The matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ are nonlinear functions of the system states

$$\boldsymbol{A}(\boldsymbol{x}) = \begin{bmatrix} L_{G_1} L_f^{r_1-1} h_1(\boldsymbol{x}) & L_{G_2} L_f^{r_1-1} h_1(\boldsymbol{x}) \\ L_{G_1} L_f^{r_2-1} h_2(\boldsymbol{x}) & L_{G_2} L_f^{r_2-1} h_2(\boldsymbol{x}) \end{bmatrix}, \quad \boldsymbol{b}(\boldsymbol{x}) = \begin{bmatrix} L_f^{r_1} h_1(\boldsymbol{x}) \\ L_f^{r_2} h_2(\boldsymbol{x}) \end{bmatrix}. \tag{4}$$

Here, L_{G_i} is the Lie derivative respect to the matrix $\boldsymbol{G}$ and $L_f^{r_1-1}$ is the Lie derivative respect to the vector $\boldsymbol{f}$ with relative degree $r_1 - 1$. For the lambda robot, the relative degree of the system outputs is $r_1 = r_2 = 2$.

4 Simulation and experimental results

To validate the designed nonlinear feedback controller, the end-effector tracks a line trajectory. Figure 2 shows the simulation and experimental results of the designed nonlinear feedback controller and its comparison to the gain scheduling controller that was presented in [4].

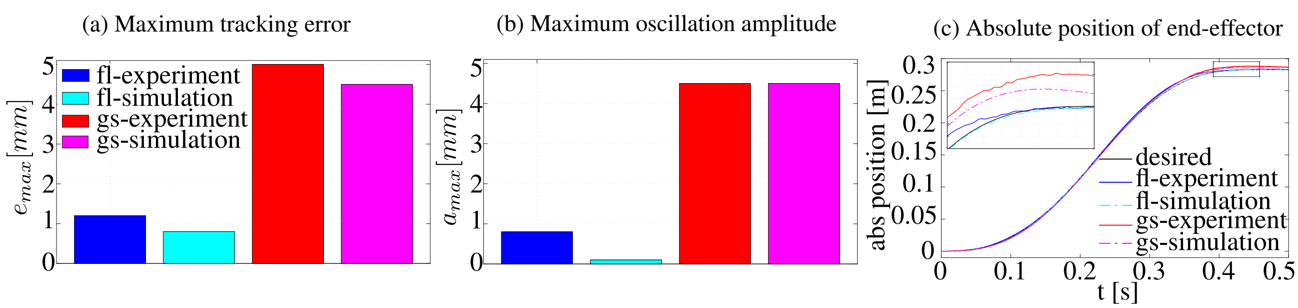

Figure 2: Simulation and experimental results for tracking a line trajectory using a feedback linearization controller (fl) and a gain-scheduling controller (gs).

The simulation and experimental results in Figure 2 show that the designed feedback linearization controller tracks the trajectory with high accuracy. The presented nonlinear controller based on the model in [1] reduces the trajectory tracking error about 76% and the end point error about 90% in simulation and experimental tests that are shown in Figures 2a and 2c. On the other hand, the oscillation amplitude in simulation and experiment is decreased about 79% as shown in Figure 2b.

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