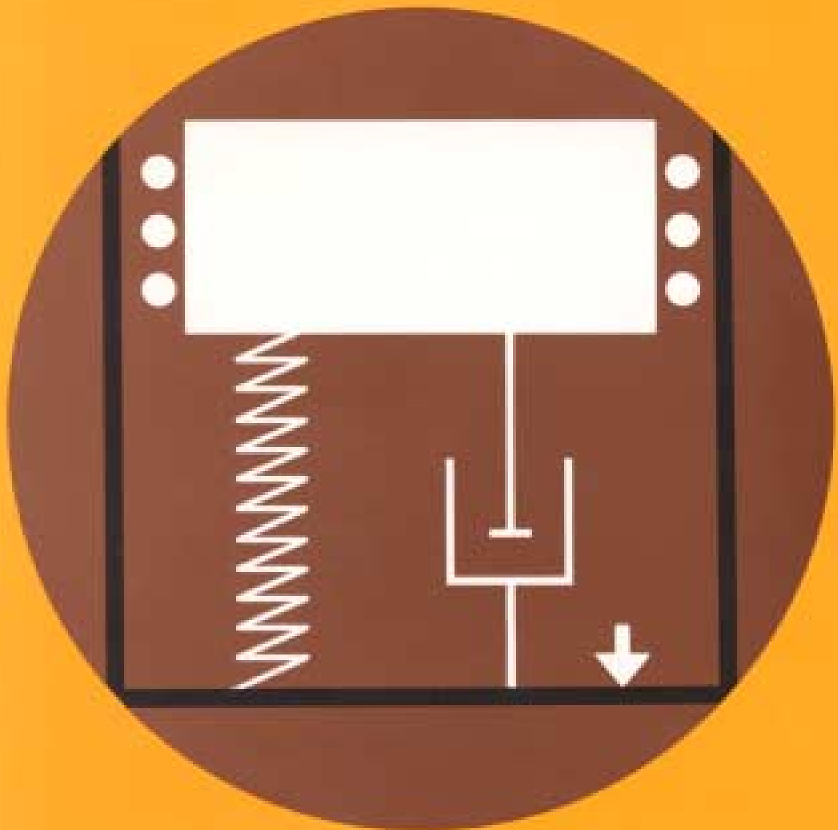
Machine Dynamics Problems

Vol. 20, 1998

Warsaw University of Technology

ISSN 0239-7730



Institute of Machine Design Fundamentals

Machine Dynamics Problems
Vol. 20, 1998, 209–219

Modeling of a Railway Wheelset as a Rotating Elastic Multibody System

Thomas Meinders
University of Stuttgart

Abstract

This paper presents a model of a railway wheelset based on the theory of elastic multibody systems. The motion of the ICE wheelset with 4 disk brakes, considering the structural elasticity, is described with a floating reference frame and superimposed elastic deformations linearized with respect to the reference frame. The governing equations of motion are generated in a symbolic form using the symbolic formalism NEWEUL. In order to compute the time-invariant system matrices describing the elastodynamical behavior of the elastic body, a preprocessor is used generating these terms in a standardized object oriented data model. To increase computational efficiency, condensation techniques are applied and 7 modes computed from a finite element model of the wheelset by ANSYS are chosen to describe the elastic behavior. A free vibration analysis at different speeds has been carried out for the rotating wheelset in order to investigate the wheelsets sensitivity to vibrations in the medium frequency range, i.e. 30 to 300 Hz. The results indicate that the wheelset shows the characteristic behavior of an elastic rotor. Subsequently simulations with the model extended by static and dynamic unbalances have been carried out. The results obtained by this analysis proves that there is no substantial coupling between torsional and bending modes without rail-wheel contact.

1. Introduction

Taking advantage of increased computational power, computer simulations are nowadays applied in the early stage of design process for new systems. During the last decades the multibody systems approach has evolved to a well known and approved method in vehicle system dynamics (Popp and Schiehlen, 1993). Applying this method to railway vehicles it is possible to obtain a good insight in the vehicle's stability and ride comfort. For recently appeared wear phenomena of the German high speed train ICE (Meinke et al., 1995) though, structural vibrations of the wheelset in the so-called medium-frequency range between 30–300 Hz seem to have a major impact. Therefor the use of rigid body systems simulation is not sufficient any more. In order to consider the elastodynamical behavior of the wheelset in addition to the rigid body motion, the method of elastic multibody systems is applied.

Analyzing elastic multibody systems the following modeling techniques can be distinguished:

- Finite element approaches using a nonlinear formulation and absolute coordinates, e.g. Cardano and Geradin (1988).
- Multibody approaches assuming a large gross motion and superimposed small elastic deformations. For the discretization of the elastic body either local (finite element method) or global shape functions can be used; the deformation is described by space dependent mode shapes and time dependent modal coordinates (modal approach), see Shabana (1989).

Due to the efficiency of the multibody approach from the computational point of view, this method is used for the modeling of the wheelset presented in this paper. Since the structure of the wheelset is rather complex, the body is discretized using local shape functions applying the finite element method.

2. Modeling of elastic multibody systems

The basic idea modeling elastic multibody systems is to describe the motion of a material point as a sum of a large gross motion (often referred to as the "rigid body" motion) and a small elastic deformation. This is done by introducing a moving reference frame that is subject to large translational and rotational motions. The position of the reference frame is given by the position vector $\mathbf{r}_j(t)$ and the rotation matrix $\mathbf{S}_j(t)$ relative to an inertial frame, see Figure 1.

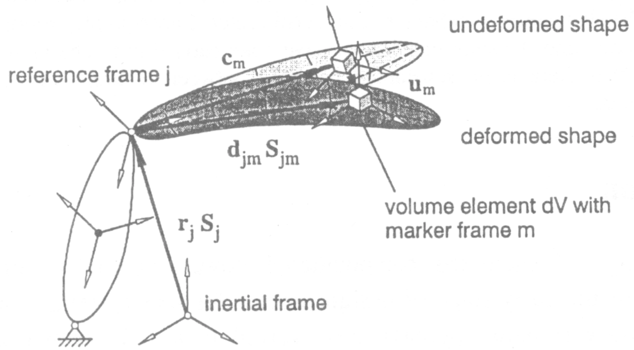


Fig. 1. General motion of an elastic body

The elastic deformations of the body, that are considered to be small, are described by a displacement vector $\mathbf{d}$ and a rotation matrix $\tilde{\vartheta}$ linearized with respect to the reference frame. The position of a volume element dV with respect to the reference frame is then described by the vector $\mathbf{c}$ to the volume element in the undeformed position and the displacement field $\mathbf{u} = \mathbf{u}(\mathbf{c}, t)$ of the elastic body, such that:

$$\mathbf{d}(\mathbf{c},t) = \mathbf{c} + \mathbf{u}(\mathbf{c},t) \tag{1}$$

In order to minimize the number of degrees of freedom of the system, a modal approach is used for the displacement field. The resulting expression for the displacement vector $\mathbf{d}$ is a linear combination of a constant mode shape matrix $\mathbf{\Phi}$, containing the selected deformation modes, and the vector $\mathbf{q}$ of generalized elastic coordinates, which are functions of time only:

$$\mathbf{u}(\mathbf{c},t) = \mathbf{\Phi}(\mathbf{c})\mathbf{q}(t) \tag{2}$$

The orientation of the marker frame relative to the reference frame is described by a rotational tensor $\mathbf{S}_{jm}(\mathbf{c},\mathbf{q})$. Assuming small deformations this tensor is composed of the [3×3] identity matrix $\mathbf{I}$ and a skew-symmetric matrix of small rotations $\overline{\vartheta} = [\alpha\beta\gamma]$ as

$$\mathbf{S}_{jm}(\mathbf{c},\mathbf{q}) = \mathbf{I} + \overline{\vartheta}(\mathbf{c},t) \tag{3}$$

The vector of small rotations can also be expressed as a linear combination of a time invariant shape matrix ψ and the vector $\mathbf{q}$ of generalized elastic coordinates as:

$$\overline{\vartheta}(c,t) = \psi(\mathbf{c})\mathbf{q}(t) \tag{4}$$

As mentioned before the discretization of the elastic body can be done using either local or global shape functions. The advantage of local shape functions (finite element method) is, that even complex geometric structures can be described. Thus, using the finite element method, the translational mode shape matrix can be expressed by

$$\psi(\mathbf{c}) = \mathbf{S}^T\mathbf{A}(c)\hat{\mathbf{S}}\mathbf{B}\mathbf{T} \tag{5}$$

where $\mathbf{A}(c)$ is the element shape function matrix, $\mathbf{B}$ is the Boolean matrix describing the assemblage of the finite elements and $\mathbf{T}$ denotes the modal matrix, summarizing eigenmodes and static modes of the elastic structure. The matrices $\mathbf{S}$ and $\hat{\mathbf{S}}$ transform the displacements from the element to the reference frame. The rotational mode shape matrix ψ is obtained in a completely analogue manner.

Before deriving the equations of motion the absolute position and orientation as well as the absolute velocities and absolute accelerations of the marker frame m attached to the volume element $\mathbf{d}V$ of the elastic body have to be formulated. Position and orientation of the frame m are described by

$$\begin{aligned} \mathbf{r}_m &= \mathbf{r}_j + \mathbf{d}_{jm} = \mathbf{r}_j + (\mathbf{c}_m + \mathbf{u}_m) \\ \mathbf{S}_m &= \mathbf{S}_j\mathbf{S}_{jm} \end{aligned} \tag{6}$$

The expressions for the absolute velocities and absolute accelerations can be derived using relative kinematics, see Melzer (1994). Applying D'Alembert's principle the equations of motion of an elastic multibody system can then be written as

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{y}}(t) + \mathbf{k}_c(\mathbf{y},\dot{\mathbf{y}},t) + \mathbf{k}_i(\mathbf{y},\dot{\mathbf{y}}) = \mathbf{q}_f(\mathbf{q},\dot{\mathbf{q}},t) \tag{7}$$

with the mass matrix **M**, the vector of generalized gyroscopic forces $\mathbf{k}_c$, the vector of elastic stiffness forces $\mathbf{k}_i$ and the vector of generalized applied forces $\mathbf{q}_f$. The vector **y** of generalized coordinates of the system comprises the coordinates $\mathbf{y}_s$ for the "rigid body" motion as well as the elastic coordinates. Thus, the vector **q** becomes a subset of the generalized coordinates **y**, such that

$$y(t) = \begin{bmatrix} y_s(t) \\ q(t) \\ \vdots \end{bmatrix} \tag{8}$$

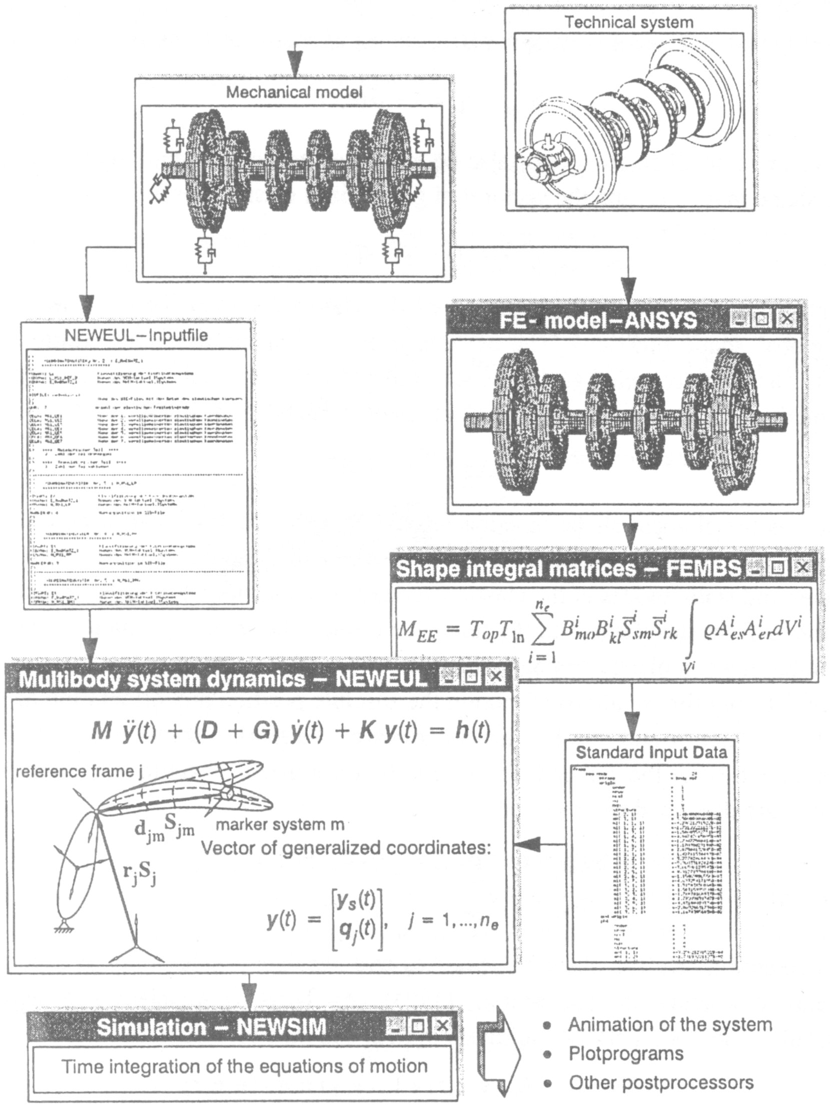


Fig. 2. Scheme of a dynamic analysis of an elastic system

The mass matrix and the vector of generalized gyroscopic forces are computed evaluating various volume integrals that depend on the elastic coordinates. Since small deformations are assumed these volume integrals can be expanded into a Taylor series of elastic coordinates of first order. The coefficient matrices of the Taylor series, the so-called shape integrals, are calculated by numerical integration. Since the shape integrals do not depend on time, they can be computed prior to time integration by pre-processing. A detailed description of this approach can be found in Melzer (1994).

The scheme of the analysis for an elastic multibody system is presented in Figure 2. Starting up form the definition of the mechanical model of the technical system, the elastic parts of the multibody system are discretized using the finite element software ANSYS (DeSalvo and Gormann, 1989). The resulting data about the mass and stiffness matrix as well as the eigenmodes of the elastic body is used by the preprocessor FEMBS (Wallrapp and Sachau, 1994) to compute the matrix describing its elastodynamical behavior. These terms are stored in a standarized format (SID) described by Wallrapp (1993). The equations of motion are generated by NEWEUL, a symbolical multibody formalism (Schiehlen, 1994). Reading the input-file defining the topological structure of the multibody system and the SID-file containing the information about the elastic body, NEWEUL yields mixed symbolic numerical equations. The simulation of the system can be carried out using standard time integration techniques.

3. Model of the wheelset

The wheelset modeled in this paper is a wheelset of the German highspeed train Inter City Express ICE. Due to the high operation speed of the ICE the wheelset is equipped with four disk brakes.

Since the description of the discretized structure of the wheelset for the finite element software ANSYS is a rather complex thing to do, it is advisable to use a parametric description for the geometry. Doing so any later changes or corrections to geometric parameters are easy to implement because the files describing the discretized structure of the wheelset can be left unchanged. Thus the model for the ICE wheelset is described by a set of 54 geometric parameters. For more information about the wheelset's data, refer to Meinders (1997).

Making use of the wheelset's symmetry only one side of the axle, two disk brakes and a wheel have to be described in detail. 2D-plane elements are used for this step which are replaced by volume elements (SOLID73 of the ANSYS library) in the next procedure rotating the 2D-structure about its principle symmetry axes. Since some of the nodes in this structure are later used to connect the finite element model to the multibody system by so-called marker coordinate systems, these nodes need to have all 6 degrees of freedom. The entire wheelset is finally obtained by reflecting the structure at the mid axes. The resulting model has a total of 13331 elements and 17904 nodes.

For the investigation of the wear phenomena – the so-called unround wheels (Zacher, 1996) – the medium frequency range up to 300 Hz is of interest. Consequently a modal analysis of the unsupported wheelset has been carried out for these frequencies. As a result the following 10 dynamic eigenmodes have been identified as listed in Table 1:

Table 1. Eigenfrequencies of the unsupported wheelset in the frequency range up to 300 Hz

Eigenfrequency	Eigenmode
82.5 Hz	1. antisymmetric torsional mode
84.6 Hz 84.6 Hz	1. symmetric bending mode (vertical) 1. symmetric bending mode (horizontal)
131.8 Hz 131.8 Hz	1. antisymmetric bending mode (vertical) 1. antisymmetric bending mode (horizontal)
188.5 Hz 188.5 Hz	2. symmetric bending mode (vertical) 2. symmetric bending mode (horizontal)
234.8 Hz	1. antisymmetric umbrella mode
261.2 Hz	1. symmetric torsional mode
296.1 Hz	1. symmetric umbrella mode

A selection of eigenmodes is depicted in Figure 3.

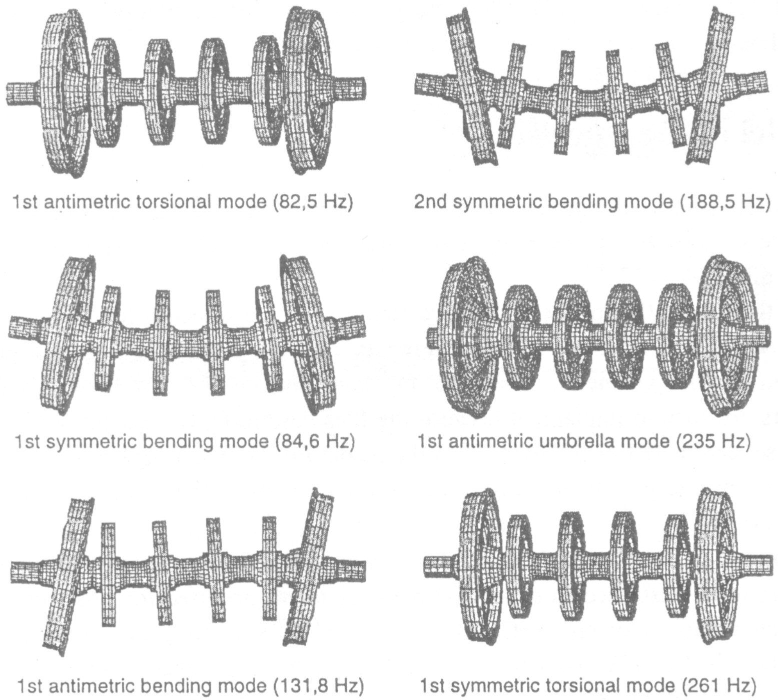


Fig. 3. Eigenmodes of the wheelset in the frequency range up to 290 Hz

The first elastic eigenmode of the wheelset at 82.5 Hz (first antisymmetric torsional mode) is characterized by a torsional motion of one side of the wheelset against the other with a

vibration node between the two inner disk brakes. Up to the first antisymmetric bending mode the deformation of the disk brakes and wheels is small enough, so that they could also be modeled as rigid bodies. This is not true any more for the second symmetric bending mode at 188,5 Hz, where the wheel disk shows a considerable deformation. Since the first antisymmetric and symmetric umbrella mode as well as the first symmetric torsional mode can hardly be excited by unbalances, they are not considered for the following investigation without wheelrail contact. Thus a total of 7 eigenmodes including the first antimetric torsional mode and the first three bending modes (double eigenmodes) are chosen as generalized elastic coordinates for the elastic multibody system.

To generate the input data for the FEM/MBS interface program FEMBS an ANSYS superelement has to be created, applying the Guyan reduction, see Guyan (1965). This requires the definition of so-called master degrees of freedom which can be selected both by the program ANSYS and the user. The master degrees of freedom should include the degrees of freedom of the nodes that will be later used as marker coordinate systems to attach other bodies, apply forces of simply for observation purposes. Using FEMBS these marker nodes have to be selected together with the de-sired eigenmodes, such that the SID data of the elastic wheelset is obtained.

The resulting model up to this point is entirely symmetric to its rotational axes. For the simulation though, the excitation due to static and dynamic unbalances is of interest. Consequently these terms are added to the system using mass points for static unbalances and massless rigid bodies with products of inertia for dynamic unbalances. The way these extra bodies are attached to the elastic body is presented in Figure 4.

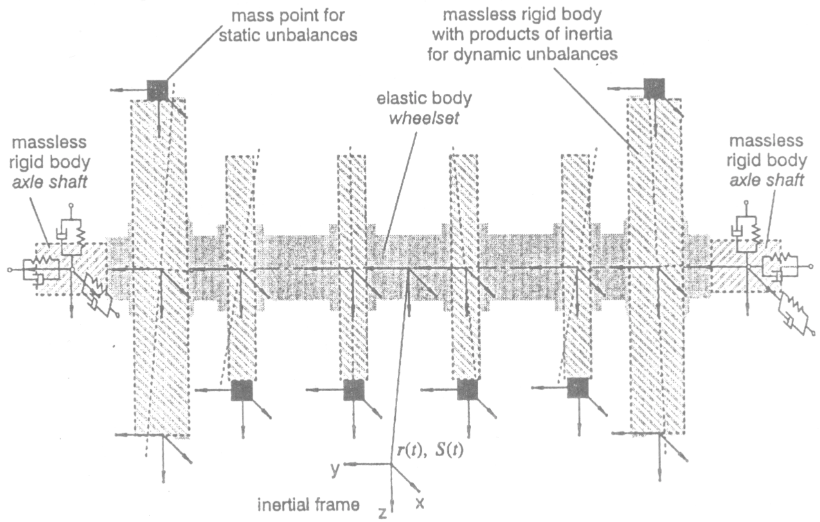


Fig. 4. Entire model of the wheelset with static and dynamic unbalances

Thus, the entire simulation model consists of the elastic wheelset, 6 rigid bodies representing the dynamic unbalances of the wheels and disk brakes, 6 masspoints modeling the static unbalances and the springs and dampers modeling the primary suspension attached to the shaft of the axle. The data for the unbalances is taken from Meinke (1995). The position of the static unbalances corresponds to the mounting prescriptions of the Deutsche Bahn AG for wheels and disk brakes on the axle. As also shown in Figure 4 the static unbalances of the wheels are twisted by 180° degrees against the unbalances of the disk brakes.

For generating the equations of motion the structure of the entire model is defined in the NEWEUL input data file. The set of generalized coordinates includes 5 variables, describing the wheelset's reference frame and 7 elastic coordinates for the selected modes. The rotation of the wheelset about its principle axes is a given motion. Thus the system has a total of 12 degrees of freedom. The complex wheel-rail contact is not yet implemented in this model; as a consequence the wheelset is only exposed to the excitation of the unbalances.

4. Eigenbehavior of the rotating wheelset

Analyzing the finite element model of the wheelset a modal analysis of the nonrotating free structure has been carried out. Since the considered ICE wheelset is running at high speed it is also of interest how the eigenbehavior depends on its rotational speed.

For the solution of this eigenvalue problem the linearized equations of motion are considered. Solving the characteristic equation yields the eigenvalues. The resulting eigenvalues of the rotating wheelset depending on the rotary frequency are presented in Figure 5. The highest rotary frequency of $\omega/2\pi$ = 30 Hz used for this investigation, corresponds to a speed of about 300 km/h.

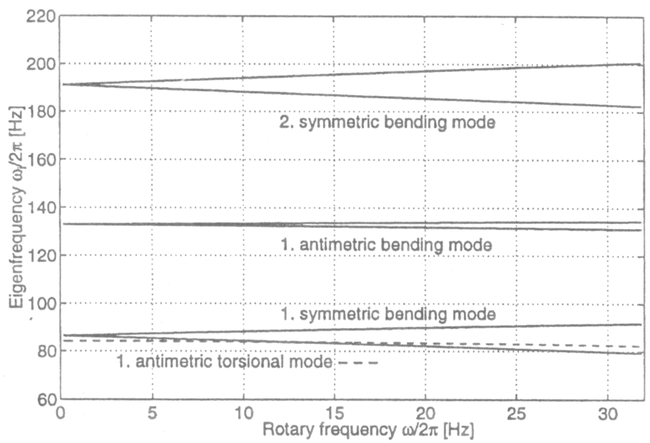


Fig. 5. Eigenbehavior of the wheelset depending on its rotary frequency

From the diagram it is clear that the eigenfrequencies of the bending modes split up into a branch with increasing eigenfrequencies and another with decreasing eigenfrequencies. The upper branch is also referred to as forward whirl, the lower branch as backward whirl. This phenomena, that is also called bifurcation of eigenfrequencies, clearly shows the characteristic behavior of an elastic rotor. The torsional mode though is not depending on the rotary frequency.

These results demonstrate that the gyroscopic terms should not be neglected for the ICE wheelset.

5. Simulation

Using standard time integration techniques, simulation of the entire model including static and dynamic unbalances have been carried out. The numerical values for the unbalances were taken from Meinke (1995), e.g. for the static unbalances of the disk brakes $U = 16gm$ and $U = 50gm$ for the wheels.

Rather than looking at the overall motion of the wheelset it is more informative to consider the share of the elastic coordinates. Figure 6 represents the results of the modeled wheelset excited by the described unbalances. The rotary frequency for this investigation is 5 Hz.

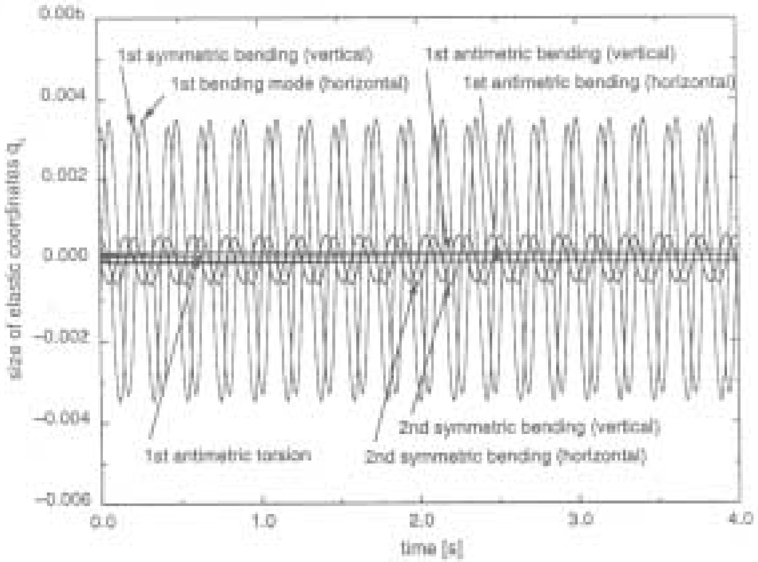


Fig. 6. Share of the elastic coordinates at the overall motion ($\omega/2\pi$ = 5 Hz)

It is recognizable that only four of seven generalized elastic coordinates considered in the model have a noticeable share at the overall motion, that is the 1st and 2nd symmetric bending mode, whose eigenforms are in correspondence with the distribution of the static unbalances. Reminding the representation of the 1st antisymmetric bending mode in Figure 3 the negligible share of its generalized coordinates is understandable due to the static unbalances.

Under certain conditions also unbalance-caused coupled bending/torsional vibrations can occur. For the wheelset modeled in this paper the vanishing share of the first antisymmetric torsional mode proves that this coupling does not appear. This is due to the fact that the wheel-rail contact has not yet been implemented in this model.

6. Concluding remarks

The elastic multibody systems approach assuming large gross motion and superimposed elastic deformations has been applied to the wheelset of the German highspeed train ICE. The body of the wheelset has been discretized using the finite element method. Due to the complex structure of the wheelset a parametric representation was introduced such that changes to the geometry can be handled easily. Furthermore the model was extended by static and dynamic unbalances. The numeric-symbolic equations of motion have been generated using the multibody system software NEWEUL.

Analyzing the system, the eigenmodes and eigenfrequencies of the finite element model in the medium frequency range up to 300 Hz have been investigated. The linearized equations of motion have been considered to study the behavior of the body at high angular velocities. The bifurcation of the wheelset's bending eigenfrequencies proves that the wheelset running at high speeds has to be treated like a rotor. Characteristic time integration simulations have been carried out analyzing the influence of the unbalances. The results demonstrate that there is no relevant coupling between the bending and torsional modes without wheel-rail contact.

Acknowledgements

This work was supported by the Deutsche Forschungsgemeinschaft DFG.

References

Cardona, A., Geradin, M., 1988, A beam finite element non-linear theory with finite rotation, *International Journal for Numerical Methods in Engineering* **26** (11), 2402–2438.

DeSalvo, G., Gorman, R., 1989, *ANSYS User's Manual*, Swanson Analysis Systems, Houston, Pensylvania.

Guyan, R., 1965, Reduction of stiffness and mass matrix, *AIAA Journal*, **3**, No. 2.

Meinders, T., 1997, *Rotordynamik eines elastischen Radsatzes*, Zwischenbericht ZB-94, Institut B für Mechanik, Universität Stuttgart.

Meinke, P., Meinke, S., Szolc, T., 1995, On Dynamics of Rotating Wheelset/Rail Systems in a Medium Frequency Range, in R. Bogacz, G.P. Ostermeyer & K. Popp (eds.), *Dynamical Problems in Mechanical Systems IV, Proceedings of the 4th Polish-German Workshop*, IPPT PAN Warsaw, 233–244.

Melzer, F., 1994, Symbolisch-numerische Modellierung elastischer Mehrkörpersysteme mit Anwendung auf rechnerische Lebensdauervorhersagen, *Fortschrittsberichte VDI-Zeitschrift*, Reihe 20, **139**, VDI-Verlag, Düsseldorf.

Popp, K., Schiehlen, W., 1993, *Fahrzeugdynamik*, Stuttgart, Teubner.

Shabana, A.A., 1989, *Dynamics of Multibody Systems*, Wiley, New York.

Schiehlen, W., 1994, Symbolic Computations in Multibody Systems, *Computer-Aided Analysis of Rigid and Flexible Mechanical Systems*, M. F. O. S. Pereira and J. A. C. Ambrosio, (eds.), Kluwer Academic Publishers, Dordrecht, 101–136.

Wallrapp, O., 1993, Standard Input Data of Flexible Members in Multibody Systems, *Advanced Multibody System Dynamics – Simulation and Software Tools*, W. Schiehlen, ed., Kluwer Academic Publishers, Dordrecht, 445–450.

Wallrapp, O., Sachau, D., 1994, Space Flight Dynamic Simulation Using Finite Element Analysis Results in Multibody Codes, *Proceedings of the 2nd Int. Conf. Computational Structure Technology*, Athens, Greece.

Zacher, M., 1996, Unrunde Räder und Oberbausteifigkeit, *ETR 45*, **10**, 605–609.

Thomas Meinders, M.Sc.
University of Stuttgart
Pfaffenwaldring 7
Germany